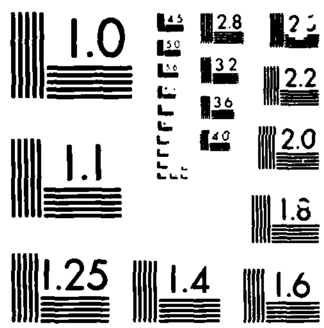
AD-A167 531 UPPER BOUNDS ON POISSON TAIL PROBABILITIES(U) WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER P W GLYNN APR 86 MRC-TSR-2931 DAAG29-80-C-0041 1/1

UNCLASSIFIED F/G 12/1 NL



MICROCOP CHART

(2)

AD-A167 531

MRC Technical Summary Report #2931

UPPER BOUNDS ON POISSON TAIL PROBABILITIES

Peter W. Glynn

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

April 1986

(Received February 26, 1986)

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

UNIVERSITY OF WISCONSIN - MADISON
MATHEMATICS RESEARCH CENTER

UPPER BOUNDS ON POISSON TAIL PROBABILITIES

Peter W. Glynn

Technical Summary Report #2931

April 1986

ABSTRACT

Upper bounds on the left and right tails of the Poisson distribution are given. These bounds can be easily computed in a numerically stable way, even when the Poisson parameter is large. Such bounds can be applied to variate generation schemes and to numerical algorithms for computing terminal rewards of uniformizable continuous-time Markov chains.

AMS (MOS) Subject Classifications: 60E15, 60F10, 60J25, 65C10, 65B10

Key Words: Bounds, Poisson distribution

Work Unit Number 5 - Optimization and Large Scale Systems

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant Nos. ECS-8404809 and DMS-8210950, Mod. 4.

SIGNIFICANCE AND EXPLANATION

Let $p(k) = \exp(-\lambda)\cdot\lambda^k/k!$ $(k = 0,1,\ldots)$ be the Poisson mass function. In a variety of application contexts, it is necessary to compute infinite sums involving these probabilities. For example, such sums occur naturally in numerical algorithms developed for Poisson variate generation purposes and for computing terminal rewards of uniformizable continuous-time Markov chains. From a practical standpoint, it is necessary to truncate these infinite sums after a finite number of terms. Development of a priori error bounds on the error incurred by this kind of truncation requires bounds on the left and right tails of the Poisson distribution; such bounds are given here. These bounds are easily computable in a numerically stable way, even when the Poisson parameter λ is large.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

UPPER BOUNDS ON POISSON TAIL PROBABILITIES

Peter W. Glynn

1. INTRODUCTION

Let $p(k) = \exp(-\lambda)\cdot\lambda^k/k!$ $(k = 0,1,\ldots)$ be the Poisson mass function. Our goal in this paper is to provide upper bounds on the left and right tail probabilities, which are defined respectively by the formulae

$$P(n) = \sum_{k=0}^{n} p(k)$$

$$\overline{P}(n) = \sum_{k=n}^{\infty} p(k) \quad .$$

Such bounds have application in several numerical methods arising from the analysis of stochastic systems.

APPLICATION 1: To generate variates from the clipped Poisson distribution, it is common to compute a table of the Poisson distribution function (see Bratley, Fox, and Schrage (1983), p. 170-171 and p. 334-335). The numerical computation of the table requires truncating the Poisson tail, thereby introducing numerical error. In order to bound the numerical error, it is necessary to have a priori bounds on the probability mass of the truncated tails (see Fox and Glynn (1986)).

APPLICATION 2: Let $X = \{X(t) : t \geq 0\}$ be a uniformizable continuous-time Markov chain with generator Q. Given that f is a real-valued function defined on the state space S of X, one is often interested in numerically determining the terminal reward $r = Ef(X(T))$. The parameter r can be computed as

$$r = \sum_{k=0}^{\infty} e^{-\alpha T} \frac{(\alpha T)^k}{k!} Ef(Y(k)) \tag{1.1}$$

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant Nos. ECS-8404809 and DMS-8210950, Mod. 4.

for $\alpha \geq \Lambda = \sup\{-Q_{xx} : x \in S\}$, where $Y = \{Y(k) : k \geq 0\}$ is an appropriately defined discrete-time Markov chain living on S. Gross and Miller (1984) have suggested numerical algorithms based on the representation (1.1). Of course, from a numerical standpoint, it is necessary to truncate the infinite sum appearing in (1.1) at some finite quantity, say m. The absolute error introduced by truncating (1.1) at m is given by

$$\varepsilon(\alpha,f) = \left| \sum_{k=m+1}^{\infty} e^{-\varphi t} \frac{(\alpha t)^k}{k!} Ef(Y(k)) \right| .$$

Let $\|f\| = \sup |f(x)| : x \in S$. By observing that $|Ef(Y(k))| \leq \|f\|$ for $k \geq 0$, it follows that $\varepsilon(\alpha,f) \leq \|f\| \cdot \overline{P}(m+1)$; thus, for bounded functions, an explicit a priori error bound can be calculated, provided that $\overline{P}(m+1)$ can be bounded.

Incidentally, by letting $f(\cdot) = K$, one finds that

$$\sup\{\varepsilon(\alpha,f) : \|f\| \leq K\} = K \cdot \overline{P}(m+1) \quad ;$$

this suggests that one should choose α so as to minimize $\overline{P}(m+1)$. Recall that $\overline{P}(m+1) = P\{N(\alpha T) > m\}$, where $N(\cdot)$ is a unit intensity Poisson process. Since $N(\cdot)$ has non-decreasing paths, $\overline{P}(m+1)$ must be non-decreasing in α. Hence, the best choice of α for minimizing $\overline{P}(m+1)$ is $\alpha = \Lambda$.

APPLICATIONS 3: The steady-state distribution of the $M/M/\infty/\infty$ infinite-server queue with arrival rate λ and service rate μ is Poisson with parameter λ/μ. An upper bound on $\overline{P}(n)$ yields an upper bound on the long-run probability that such a queueing system contains n or more customers.

In the above applications, an upper bound on the tails is used to determine $n_1(\varepsilon)$, $n_2(\varepsilon)$ for which $P\{n_1(\varepsilon) \leq N(\lambda) \leq n_2(\varepsilon)\} \geq 1 - \varepsilon$, where ε is a prescribed error tolerance. (In Application 2, one would generally take $n_1(\varepsilon) = 0$.) One can argue that a straightforward method exists for choosing such an $(n_1(\varepsilon), n_2(\varepsilon))$ pair. Choose $n_1(\varepsilon) = 0$ and let $n_2(\varepsilon)$ be the first m for which $P(m) \geq 1 - \varepsilon$; the latter operation can be done numerically by successively adding the mass probabilities $p(k)$. This method has two disadvantages. Firstly, the computation of $n_2(\varepsilon)$ requires programming set-up time and expense. Secondly, for large λ, the computation of the mass probabilities $p(k)$ involves significant numerical underflow-overflow problems; overcoming these difficulties is non-trivial (see Fox and Glynn (1986)).

On the other hand, for large λ, the central limit theorem applies to yield

$$(1.2)\qquad \begin{aligned} &P\{\lambda + z_1\lambda^{1/2} \le N(\lambda) \le \lambda + z_2\lambda^{1/2}\} \\ &\quad \to \Phi(z_2) - \Phi(z_1) \end{aligned}$$

as $\lambda \to \infty$, where $\Phi(x) = \int_{-\infty}^{x} \phi(t)dt$ and $\phi(x) = (2\pi)^{-1/2} \exp(-x^2/2)$. The problem in applying (1.2) (together with a table of the normal distribution function) to obtain $n_1(\varepsilon)$ and $n_2(\varepsilon)$ is that (1.2) is only true in the limit. For finite λ, (1.2) gives no usable information. An obvious refinement would be to combine (1.2) with the Berry-Esseen theorem (see p. 542 of Feller (1971)). This, however, leads nowhere since the Berry-Esseen error bound is independent of z_1 and z_2 and consequently gives no usable upper bound on the Poisson tail probabilities.

Our error bounds avoid these difficulties. Our first result bounds the tail probabilities in terms of the mass function.

(1.3) PROPOSITION. Assume $\lambda > 0$.

i.) If $0 \le n < \lambda$, then

$$P(n) \le p(n) \cdot (1 - (n/\lambda))^{-1}$$

ii.) If $n > \lambda - 1$ and $m \ge 1$, then

$$\overline{P}(n) \le (1 - (\frac{\lambda}{n+1})^m)^{-1} \cdot \sum_{k=n}^{n+m-1} p(k).$$

The following corollary is immediate.

(1.4) COROLLARY. (i) $\lim_{\lambda \to \infty} P(n)/p(n) = 1$

(ii) $\lim_{n \to \infty} \overline{P}(n)/p(n) = 1.$

According to the second part of our corollary, virtually all the mass of the Poisson right tail $\overline{P}(n)$ sits at the point n. A further consequence of (1.3) ii.) (with $m = 1$)

is that for $\alpha > 0$,

(1.5) $$1 < \frac{\overline{P}(\lceil\lambda(1+\alpha)\rceil)}{p(\lfloor\lambda(1+\alpha)\rfloor)} < 1 + 1/\alpha \quad ,$$

which implies that

(1.6) $$\lim_{\lambda\to\infty} \frac{1}{\lambda}\,\overline{P}(\lceil\lambda(1+\alpha)\rceil) = \lim_{\lambda\to\infty} \frac{1}{\lambda}\, p(\lceil\lambda(1+\alpha)\rceil)$$
$$= \alpha - (1+\alpha)\ln(1+\alpha) \quad ;$$

(1.6) is a statement of Chernoff's large deviations theorem (see Bahadur (1971), p. 6-9) specialized to the Poisson distribution. Thus, (1.5) may be viewed as a refinement of Chernoff's result.

The bounds given by (1.3) can be used directly when λ is small, since $p(k)$ can then be evaluated without numerical difficulties arising. If λ is large, it is useful to have error bounds on the mass function itself. Let $a = \lfloor\lambda\rfloor$.

(1.7) PROPOSITION. Let $\lambda, n \geq 1$. Then,

i.) $p(a-n) \leq (2\pi a)^{-1/2}\exp(-n(n-1)/2\lambda)$

ii.) $p(a+n) \leq (2\pi a)^{-1/2} \cdot \exp\left(\frac{-n(n-1)}{2\lambda} + \frac{(n-1)n(2n-1)}{12\lambda^2}\right)$.

The normal approximation (1.2) indicates that the choices $n_1(\varepsilon)$, $n_2(\varepsilon)$ will be of order $\lambda^{1/2}$ from λ. Thus, the values of n appearing in (1.3), which are of most concern, are those within order $\lambda^{1/2}$ from λ. Consequently, if λ is large, the factors n/λ and $\lambda/(n+1)$ appearing in (1.3) i.) and ii.) respectively will be close to one, causing the bounds to blow up. Thus, more refined bounds are needed when λ is large.

(1.8) THEOREM. Suppose $\lambda \geq 2$. Let $\overline{\Phi}(x) = \Phi(-x)$ be the standard normal right tail probability.

i.) If $n \geq 2$,

$$P(a-n) \leq \exp(1/8\lambda)\cdot(1 + \tfrac{1}{\lambda})\cdot\overline{\Phi}\left(\frac{n-3/2}{\lambda^{1/2}}\right)$$

ii.) If $2 \leq n \leq (\lambda+3)/2$,

$$\overline{P}(a+n) \leq \exp(1/8\lambda)\cdot(1 + \tfrac{1}{\lambda})\cdot\sqrt{2}\cdot\left(1 - \left(\tfrac{\lambda}{a+n+1}\right)^a\right)^{-1}\cdot\overline{\Phi}\left(\frac{n-3/2}{(2\lambda)^{1/2}}\right).$$

An obvious limitation of (1.3) ii.) is the restriction on the size of n. However, since one expects to use these bounds only for large λ (in which case n will tend to be of order $\lambda^{1/2}$), this restriction is not serious. To avoid the computation of the numerically hard-to-compute reciprocal factor in (1.8) ii.), we also offer the following bound.

(1.9) PROPOSITION. Suppose $\lambda > 2$. For $2 < n < (\lambda+3)/2$,

$$\overline{P}(a+n) < \exp(1/16)\cdot\left(1 + \frac{1}{\lambda}\right)\cdot\sqrt{2}\cdot\left(1 - \exp\left(-\frac{2n}{9}\right)\right)^{-1}\cdot\Phi\left(\frac{n-3/2}{(2\lambda)^{1/2}}\right) .$$

Note that all the terms appearing in the upper bounds of (1.8) i.) and (1.9) can be easily computed in a numerically stable fashion. These bounds should therefore be suitable for the applications previously described.

2. PROOFS

PROOF OF PROPOSITION 1.3: Observe that

$$P(n) = p(n)\cdot(1 + \sum_{k=0}^{n-1} \frac{n(n-1)\cdots(k+1)}{\lambda^{n-k}})$$

$$\leq p(n)\cdot(1 + \sum_{k=0}^{n-1} (\frac{n}{\lambda})^{n-k})$$

$$\leq p(n)\cdot \sum_{k=0}^{\infty} (\frac{n}{\lambda})^k = p(n)\cdot(1 - \frac{n}{\lambda})^{-1} \quad .$$

For $\overline{P}(n)$, it is evident that

$$\overline{P}(n) = \sum_{k=n}^{n+m-1} p(k) + \lambda^m \cdot \sum_{k=n}^{\infty} p(k)\cdot\frac{k!}{(k+m)!}$$

$$\leq \sum_{k=n}^{n+m-1} p(k) + \lambda^m \cdot \sum_{k=n}^{\infty} p(k)\cdot(n+1)^{-m}$$

$$= \sum_{k=n}^{n+m-1} p(k) + (\frac{\lambda}{n+1})^m \overline{P}(n) \quad .$$

Solving for $\overline{P}(n)$ yields (1.3) ii.).

The following lemma collects a series of inequalities which we shall need for the remainder of our proofs.

(2.1) LEMMA. i.) for $y > -1$, $\ln(1+y) \leq y$

ii.) for $y \geq 0$, $-\ln(1+y) \leq -y + y^2/2$

iii.) for $y \geq 0$, $(1+y)^{1/2} \leq 1 + y/2$.

PROOF. For i.), let $g(y) = y - \ln(1+y)$ and observe that $g(0) = 0$ and $g'(y) < 0$ for $-1 < y < 0$, whereas $g'(y) > 0$ for $y > 0$. Hence, 0 is a global minimizer of $g(\cdot)$, so that $g(y) \geq g(0) = 0$ for $y > -1$. The proofs of ii.) and iii.) are similar.

PROOF OF PROPOSITION 1.7: For $1 \leq n \leq a$, we use the fact that $a \leq \lambda$ to obtain

$$p(a-n) = p(a)\cdot(\tfrac{a}{\lambda})(\tfrac{a-1}{\lambda})\cdots(\tfrac{a-n+1}{\lambda})$$

$$\leq p(a)\cdot(1)\cdot(1-\tfrac{1}{\lambda})\cdots(1-\tfrac{(n-1)}{\lambda})$$

$$= p(a)\cdot\exp(\sum_{k=0}^{n-1} \ln(1-\tfrac{k}{\lambda})) \quad .$$

By (2.1) i.), $\ln(1-k/\lambda) \leq -k/\lambda$ so

$$p(a-n) \leq p(a)\cdot\exp(-\sum_{k=0}^{n-1} k/\lambda)$$

$$= p(a)\cdot\exp(-n(n-1)/2\lambda) \quad ,$$

by a standard summation formula (see Knuth (1969), p. 55). To bound $p(a)$, we use a Stirling formula-type inequality (see Feller (1968), p. 54)

$$a! > (2\pi a)^{1/2} a^a\cdot e^{-a}\cdot\exp(1/(12a+1))$$

which yields $(b = \lambda - a)$

$$p(a) \leq \frac{e^{-\lambda}\cdot\lambda^a}{\sqrt{2\pi a}\; a^a\cdot e^{-a}}$$

$$= \frac{1}{\sqrt{2\pi a}} e^{-b}(1+\tfrac{b}{a})^a \tag{2.2}$$

$$\leq \frac{1}{\sqrt{2\pi a}} e^{-b}\cdot e^{b} = (2\pi a)^{-1/2} \quad ;$$

the last inequality is obtained by exponentiating both sides of $\ell n(1+b/a) \leq b/a$ (see (2.1) i.)). This proves (1.7 i.) for $n \leq a$; for $n > a$, the inequality is trite. As for $p(a+n)$, use $a \geq \lambda - 1$ to obtain

$$p(a+n) = p(a)\cdot\frac{\lambda^n}{(a+1)(a+2)\cdots(a+n)}$$

$$\leq p(a)\cdot\frac{\lambda^n}{\lambda(\lambda+1)\cdots(\lambda+n-1)}$$

$$= p(a)\cdot\exp(-\sum_{k=0}^{n-1} \ln(1+\tfrac{k}{\lambda}))$$

$$\leq p(a)\cdot\exp(-\sum_{k=0}^{n-1}\frac{k}{\lambda} + \sum_{k=0}^{n-1}\frac{k^2}{2\lambda^2}) \quad ,$$

the latter inequality by (2.1) ii.). Using standard summation formulae (see Knuth (1969), p. 55) and (2.2) gives (1.7) ii.).

PROOF OF THEOREM 1.8: By (1.7) i.),

$$P(a-n) \le (2\pi a)^{-1/2} \cdot \sum_{k=n}^{a} \exp(-\frac{k(k-1)}{2\lambda})$$

Since $g(x) = -x(x-1)/2\lambda$ is non-increasing on $[1/2,\infty)$,

$$\exp(-x(x-1)/2\lambda) \le \int_{x-1}^{x} \exp(-u(u-1)/2\lambda)du$$

for $x \ge 3/2$. Thus, if $n \ge 2$,

$$P(a-n) \le (2\pi a)^{-1/2} \exp(1/8\lambda) \cdot \int_{n-1}^{\infty} \exp(-(u-1/2)^2/2\lambda)du$$

(2.3)

$$\le \exp(1/8\lambda)(\lambda/a)^{1/2} \bar{\Phi}((n-3/2)/\lambda^{1/2}) \quad .$$

By (2.1) iii.), $(\lambda/a)^{1/2} \le 1 + b/2a$. For $\lambda \ge 2$, $\lambda \le \lfloor\lambda\rfloor + 1 \le \lfloor\lambda\rfloor + 2 - 2/\lambda = \lfloor\lambda\rfloor + (2/\lambda)\cdot(\lambda-1) \le \lfloor\lambda\rfloor + (2/\lambda)\cdot\lfloor\lambda\rfloor$ so that $b/2a \le 1/\lambda$; substituting into (2.3) yields (1.8) i.).

For (1.8) ii.), we first use (1.3) ii.) with $m = a$ to obtain

$$\bar{P}(a+n) \le (1 - \frac{\lambda}{a+n+1})^{a})^{-1} \cdot \sum_{k=n}^{n+a-1} p(a+k) \quad .$$

For $n \le k \le n+a-1$, we have

$$-\frac{k(k-1)}{2\lambda} + \frac{(k-1)k(2k-1)}{12\lambda^2}$$

$$\le -\frac{k(k-1)}{2\lambda} \cdot \beta$$

where $\beta = 1 - (n+a)/(3\lambda) + (2\lambda)^{-1}$. By (1.7) ii.), it follows that

(2.4) $$\bar{P}(a+n) \le (1 - (\frac{\lambda}{a+n+1})^{a})^{-1} \cdot (2\pi a)^{-1/2} \cdot \sum_{k=n}^{n+a-1} \exp(-k(k-1)\beta/2\lambda) \quad .$$

As in the bound for $P(a-n)$, the latter sum is dominated by

$$\int_{n-1}^{\infty} \exp(-u(u-1)\beta/2\lambda)du$$

(2.5)

$$= \exp(\beta/8\lambda) \cdot \sqrt{\frac{\lambda}{\beta}} \cdot (2\pi)^{1/2} \cdot \bar{\Phi}(\frac{(n-3)}{2}\sqrt{\frac{\beta}{\lambda}}) \quad .$$

Now, $\beta < 1$ since $n+a > n > 2$; furthermore, since $n < (\lambda+3)/2$, it follows that $\beta > 1 - (\lambda/2 + 3/2 + \lambda)/3\lambda + (2\lambda)^{-1} = 1/2$. Hence, $\beta^{-1/2} < \sqrt{2}$, $\exp(\beta/8\lambda) < \exp(1/8\lambda)$, and

$$\bar{\Phi}(\frac{(n-3)}{2}\sqrt{\frac{\beta}{\lambda}}) < \bar{\Phi}(\frac{(n-3)}{2})\cdot(2\lambda)^{-1/2}) \quad .$$

Combining these inequalities and the previously obtained $(\lambda/a)^{1/2} < (1 + 1/\lambda)$ with (2.4) and (2.5), we get (1.8) ii.).

PROOF OF PROPOSITION 1.9: For $\lambda > 2$, $\exp(1/8\lambda) < \exp(1/16)$, so it remains only to show that

$$(1 - (\frac{\lambda}{a+n+1})^{a})^{-1} < (1 - \exp(\frac{-2n}{a}))^{-1} \quad . \tag{2.6}$$

Since $a+1 > \lambda$, it follows that $\lambda/(a+n+1) < 1 - (n/(\lambda+n))$. Now,

$$1 - \frac{n}{\lambda+n} < \exp(-n/(\lambda+n))$$

(exponentiate both sides of (2.1) i.)), so

$$(\frac{\lambda}{a+n+1})^{a} < \exp(\frac{-na}{\lambda+n}) \quad . \tag{2.7}$$

The function $f(x) = (x-1)/(x+1)$ is non-decreasing on $[0,\infty)$ so $f(x) > f(2) = 1/3$ for $x > 2$. Thus, for $\lambda > 2$, $(\lambda-1)/(\lambda+1) > 1/3$, proving that $a > (1/3)(\lambda+1)$. Hence, $a(\lambda+n)^{-1} > a(\lambda + \lambda/2 + 3/2)^{-1} > (\lambda+1)/(3\cdot(\lambda+1)\cdot 3/2) = 2/9$. Relation (2.7) then yields

$$(\frac{\lambda}{a+n+1})^{a} < \exp(-2n/9) \quad ,$$

from which (2.6) follows immediately.

REFERENCES

1. BAHADUR, R. R. (1971). Some Limit Theorems in Statistics. Society for Industrial and Applied Mathematics, Philadelphia.
2. BRATLEY, P., FOX, B. L., and L. E. SCHRAGE (1983). A Guide to Simulation. Springer-Verlag, New York.
3. FELLER, W. (1968). An Introduction to Probability Theory and its Applications, Volume I. John Wiley, New York.
4. FELLER, W. (1971). An Introduction to Probability Theory and its Applications, Volume II, John Wiley, New York.
5. FOX, B. L. and P. W. GLYNN (1986). Computing Poisson Probabilities. Technical Report, Université de Montréal.
6. GROSS, D. and D. R. MILLER (1984). The randomization technique as a modeling tool and solution procedure for transient Markov processes. Operations Research 32, 343-361.
7. KNUTH, D. E. (1969). The Art of Computer Programming, Volume 1. Addison-Wellesley, Reading, Massachusetts.

PWG/jvs

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER #2931	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Upper Bounds on Poisson Tail Probabilities		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Peter W. Glynn		8. CONTRACT OR GRANT NUMBER(s) ECS-8404809; DMS-8210950, Mod 4 DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53705		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 5 - Optimization and Large Scale Systems
11. CONTROLLING OFFICE NAME AND ADDRESS See Item 18 below		12. REPORT DATE April 1986
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Bounds, Poisson distribution

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Upper bounds on the left and right tails of the Poisson distribution are given. These bounds can be easily computed in a numerically stable way, even when the Poisson parameter is large. Such bounds can be applied to variate generation schemes and to numerical algorithms for computing terminal rewards of uniformizable continuous-time Markov chains.

END
FILMED

6-86

DTIC